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HYPERSONIC VISCOUS FLOW ON NONINSULATED FLAT PLATE[1]

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Abstract

An analytical treatment of the "strong" interaction problem in hypersonic viscous flow on a noninsulated flat plate is presented, using the method of "similar" solutions of the compressible boundary layer equations. Recent experimental data which confirm some of the theoretical results are also discussed.

Nomenclature

x, y	coordinate axes
u, v	velocity components in the x, y directions respectively
M_∞	free stream Mach number
p	pressure
a	velocity of sound
ρ	density
T	temperature

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μ	coefficient of viscosity
k	coefficient of heat conductivity
C_p	specific heat at constant pressure
R	gas constant
$H = C_pT + \frac{1}{2}u^2$	total energy
γ	ratio of specific heats
K	defined in Eq 5
G	defined in Eq 5
m	the constant parameter in Eq 1
η	similarity variable
δ	boundary layer thickness
$C_{f_\infty} = \dfrac{\mu_w\left(\dfrac{\partial u}{\partial y}\right)_w}{\frac{1}{2}\rho_\infty u_\infty^2}$	local skin friction coefficient
$C_{h_\infty} = \dfrac{k_w\left(\dfrac{\partial T}{\partial y}\right)_w}{\rho_\infty u_\infty H_\infty (G_w - 1)}$	local heat transfer coefficient
$R_{e_x} = \dfrac{\rho_\infty u_\infty x}{\mu_\infty}$	free stream Reynolds number on the basis of x
A, B, C, D	defined in Eqs 16-19
ω	exponent in the viscosity law

Subscripts "o", "1", "∞", "w" refer, respectively, to the stagnation conditions, the conditions at the edge of the boundary layer, the conditions in the free stream, and the conditions on the surface.

Introduction

The study of hypersonic viscous gas flow past a flat plate constitutes one of the best sources of information on the nature of hypersonic interaction between the viscous and inviscid effects. Serious efforts, both analytical and experimental, have recently been directed towards the understanding of this interesting problem. Shen [1] (Numbers in brackets refer to the bibliography at the end of the paper.) first attempted in 1949 an analytical estimate of the viscous effects of hypersonic flow over an insulated wedge. Shen assumed that the viscous effects are confined in the region from the body surface to the leading edge shock wave. He concluded [2] that the compressible boundary layer equations are valid, as a first approximation, in the viscous layer on the body. By a simple momentum integral analysis, it was possible to treat the shock wave and boundary layer interaction phenomena at hypersonic speeds as a complete problem. In an experimental study of the hypersonic flow on a flat plate at the Langley NACA Laboratory, probably concurrent to Shen's research, Becker [3] observed that, at $M_\infty = 6.86$, there is a significant pressure rise on the frontal portion of the plate. Bertram [4] in an attempt to interpret these experimental findings, proposed in 1952 an approximate method for determining the displacement effects and viscous drag of laminar boundary layers in two-dimensional hypersonic flow. In Bertram's work, the region behind the shock wave was treated as consisting of a viscous boundary layer and an inviscid region between the shock and the boundary layer. In 1952, Lees and Probstein [5] at Princeton gave a more rigorous treatment of Bertram's problem of weak interaction between the shock wave and the boundary layer in hypersonic flow. In the same year, the present authors presented a revised theory [6] of Shen's problem of strong interaction. In [6], it was discovered that (1) the strong interaction problem yields a pressure distribution law in the front part of a flat plate, $p \alpha x^{-\frac{1}{2}}$, (2) the boundary layer in the strong interaction region grows like $\delta \alpha x^{3/4}$, and (3) the solutions have the characteristics of "similar" solutions of the compressible boundary layer equations. These findings were later confirmed by Lees [7], who formulated an analytical theory, not using the momentum integral method, of the strong interaction problem. However, in [7], Lees retained the

assumption that the shock wave and the boundary layer are separated by an inviscid rotational flow region. In order to affect a reasonable matching between the viscous and inviscid regions, Lees had to introduce the tangent-wedge approximation [8].

In an entirely general discussion of a method of treating the compressible boundary layer equation, Stewartson [9] pointed out in 1949 the existence of "similar" solutions. Stewartson's concept was carried out in great detail by the present authors in 1952 [10]. A new approach to the problem of "similar" solutions was also presented in a recent paper [11] in which it was pointed out that Lees' results in (7) are a special case of the "similar" solutions. Thus, the treatment of the hypersonic interaction problem and the discussion on "similar" solutions are completely correlated.

It is interesting to note that the hypersonic interaction phenomena give a physically significant example of the "similar" solutions of the compressible boundary layer equations, but the most interesting point is that by the use of the "similar" solutions it is possible to extend the analytical treatment of the hypersonic interaction problem to include more general cases. For instance, for the strong interaction problem, there is not yet a satisfactory theory dealing with the case of a noninsulated flat plate. This particular problem will be treated in the present paper by the method of "similar" solutions.

The adoption of the present treatment, however, depends on two vital conditions: (1) In the case $M_\infty >> 1$, the velocity at the edge of the boundary layer is nearly constant, $u_1 \simeq u_\infty$; (2) In the frontal part of the flat plate, $p \propto x^{-1/2}$. Recent experimental studies at GALCIT [12] and Princeton [13] have confirmed these conditions. They can be readily utilized to reduce the system of partial differential equations to ordinary equations by the method of [11]. Thus, a theory of hypersonic viscous flow on a noninsulated flat plate can be presented entirely within the framework of the method of "similar" solutions.

The results of the present study are of practical importance because they give an estimate of the effects of heat transfer in the strong interaction region on a flat plate. Presumably, if sufficient knowledge of the heat transfer effects is accumulated, it will be possible eventually to control effectively the strong interaction effects.

In [11], it has been shown that if (1) the Prandtl number of the fluid is assumed unity, and (2) the viscosity of the fluid is assumed to behave like a linear function of T, i.e., $\mu = \frac{\mu_o}{T_o} T$, then the partial differential equations of the steady, two-dimensional compressible boundary layer can be transformed into a system of ordinary equations as follows:

$$KK'' + K''' = \frac{1}{m}(K'^2 - G) \tag{1}$$

$$G'' + KG' = 0 \tag{2}$$

provided that the main flow outside the boundary layer satisfies the following differential equation:

$$(a_o^2 - a_1^2)^{m-\frac{3}{2}} (a_1^2)^{\frac{1-2\gamma}{\gamma-1}-m} \frac{da_1^2}{dx} = -\frac{\gamma-1}{mp_o}\left(\frac{2}{\gamma-1}\right)^{\frac{3}{2}-m} (a_o^2)^{\frac{1}{\gamma-1}} \tag{3}$$

In Eqs 1 and 2, K and G are functions of a similar variable:

$$\eta(x, y) = \frac{1}{\sqrt{R_o \frac{\mu_o}{T_o}}} \int_o^y a_1^{m+\frac{2\gamma}{\gamma-1}} u_1^{1-m} \frac{dy}{T(x, y)} \tag{4}$$

and the prime denotes differentiation with respect to η. The physical significance of K and G is evident from the following definitions:

$$K' = \frac{u}{u_1(x)}, \qquad G = \frac{H}{H_1} \tag{5}$$

The proper boundary conditions are

$$\eta = o, \qquad K = K' = o, \qquad G = \frac{T_w}{T_o} \tag{6}$$

$$\eta \to \infty, \qquad K' = 1, \qquad G = 1 \tag{7}$$

In hypersonic flow, $M_\infty^2 >> 1$, the flow velocity behind the leading edge shock wave is not much different from the undisturbed free stream velocity, i.e.,

A-1 20

$$u_1 = u_\infty - 0\left(\frac{1}{M_\infty^2}\right) \tag{8}$$

For the "strong" interaction problem of the hypersonic viscous flow on a flat plate, if the outer edge of the boundary layer is regarded, to a first approximation, as a streamline, then it can be shown [14] that Eq 3 becomes

$$\left(\frac{p_1}{p_0}\right)^{\frac{m}{\gamma} - (m+2)} \frac{d}{dx}\left(\frac{p_1}{p_0}\right) = -\frac{\gamma}{mp_0}(a_0^2)^{\frac{\gamma+1}{\gamma-1}+m}(u_\infty^2)^{\frac{3}{2}-m} \tag{9}$$

It is concluded, therefore, that for the hypersonic boundary layer problem, the constant parameter m in Eq 1 must satisfy the following condition:

$$m = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1+n}{n}\right) \tag{10}$$

where n is an index number of the main stream pressure, viz., $\frac{p_1}{p_0} \propto x^n$. Previous calculations in [6] show that $n = -\frac{1}{2}$. Hence

$$m = \frac{\gamma}{\gamma - 1} \tag{11}$$

Eqs 1 and 2 with the boundary conditions of Eqs 6 and 7, in the case of values for m as given in Eq 11, have been integrated on a REAC machine with $\gamma = 1.4$ and $\gamma = 1.67$ for a range of values of T_W/T_O. These solutions have previously been reported in [11]. In the present paper, this data will be used to compute, for various T_W/T_O's, the hypersonic viscous flow characteristics on a flat plate in an air atmosphere or helium atmosphere. These results are invaluable in the estimation of the effectiveness of surface cooling or heating in controlling the "strong" interaction effects.

Computations in [14] show that within the framework of an approximation by the method of similar solutions, the "strong" interaction between the leading edge shock wave and the viscous boundary layer on a flat plate in hypersonic flow is characterized by the following general laws:

$$\frac{p_1}{p_\infty} = A\,\frac{M_\infty^3}{\sqrt{Re_x}} \tag{12}$$

$$M_\infty \frac{\delta}{x} = B\left(\frac{M_\infty^3}{\sqrt{Re_x}}\right)^{1/2} \tag{13}$$

$$C_{f_\infty}\sqrt{Re_x} = C\left(\frac{M_\infty^3}{\sqrt{Re_x}}\right)^{1/2} \tag{14}$$

$$C_{h_\infty}\sqrt{Re_x} = D\left(\frac{M_\infty^3}{\sqrt{Re_x}}\right)^{1/2} \tag{15}$$

where A, B, C, D are defined as follows:

$$A = \frac{3}{4}\,(\gamma - 1)\sqrt{\frac{\gamma(\gamma+1)}{2}}\int_0^\infty (G - K'^2)\,d\eta \tag{16}$$

$$B = \frac{1}{[\gamma(\gamma+1)]^{1/4}}\,\frac{2}{\sqrt{3}}\,\sqrt{\gamma - 1}\left[\sqrt{2}\int_0^\infty (G - K'^2)\,d\eta\right]^{1/2} \tag{17}$$

$$C = \frac{3}{4}\sqrt{\frac{\gamma(\gamma+1)}{2}}\left\{\frac{2}{\sqrt{3}}\sqrt{\gamma-1}\left[\frac{2}{\gamma(\gamma+1)}\right]^{1/4}\left[\int_0^\infty (G - K'^2)\,d\eta\right]^{1/2}\right\}K''(o) \tag{18}$$

$$D = -\frac{3}{8}\sqrt{\frac{\gamma(\gamma+1)}{2}}\left\{\frac{2}{\sqrt{3}}\sqrt{\gamma-1}\left[\frac{2}{\gamma(\gamma+1)}\right]^{1/4}\left[\int_0^\infty (G - K'^2)\,d\eta\right]^{1/2}\right\}\frac{G'(o)}{(G_w-1)} \tag{19}$$

The values of A, B, C, D for various T_w/T_o's, with $\gamma = 1.4$ and $\gamma = 1.67$, are presented in Figs. 1, 2, 3, and 4, respectively. They are also tabulated in Table I. It is observed that the integral

$$I = \int_0^\infty (G - K'^2)\,d\eta \tag{20}$$

Table I

VALUES OF A, B, C, D FOR VARIOUS T_w/T_o's

	γ = 1.4				γ = 1.67			
T_w/T_o	A	B	C	D	A	B	C	D
0	.1485	.3967	.2082	.1888	.2614	.4572	.2867	.2494
.2	.2316	.4954	.2793		.4029	.5675	.3946	
.6	.3773	.6323	.4118	.3105	.6596	.7261	.6015	.4199
1.0	.5140	.7381	.5488	.3732	.9207	.8577	.8199	.5042
2.0					1.506	1.097	1.361	.6832

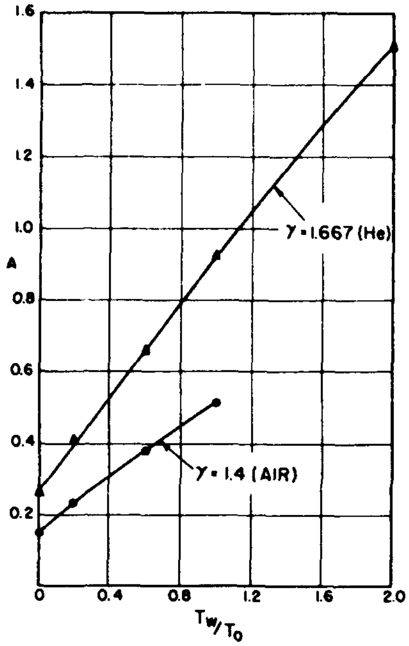
enters all the formulae for A, B, C, and D. This integral is evaluated by Simpson's rule, with $\Delta\eta = 0.1$, from the data obtained from the REAC. The values of I for various T_w/T_o's are given in Table II.

Table II

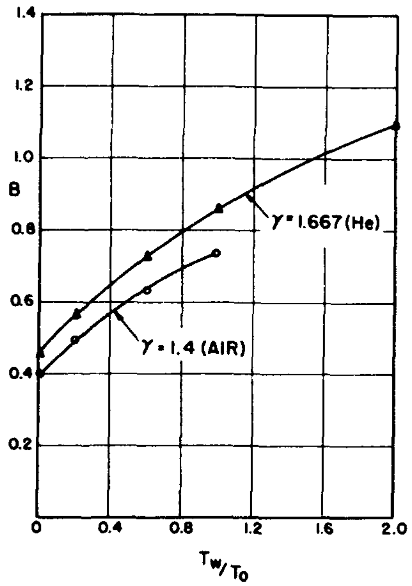
VALUES OF $I = \int_0^\infty (G - K'^2)\,d\eta$ FOR VARIOUS T_w/T_o's

γ	T_w/T_o	I
1.4	0	.3820
1.4	.2	.5956
1.4	.6	.9703
1.4	1.0	1.322
1.67	0	.3507
1.67	.2	.5404
1.67	.6	.8848
1.67	1.0	1.235
1.67	2.0	2.020

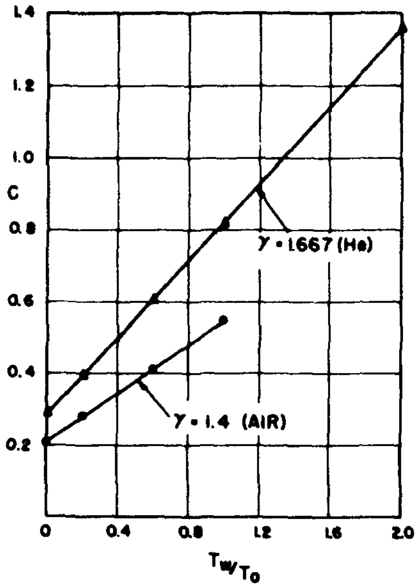
In Figs. 1-4, $T_w/T_o = 1.0$ corresponds to an insulated surface; $T_w/T_o < 1.0$, a cooled surface; and $T_w/T_o > 1.0$, a



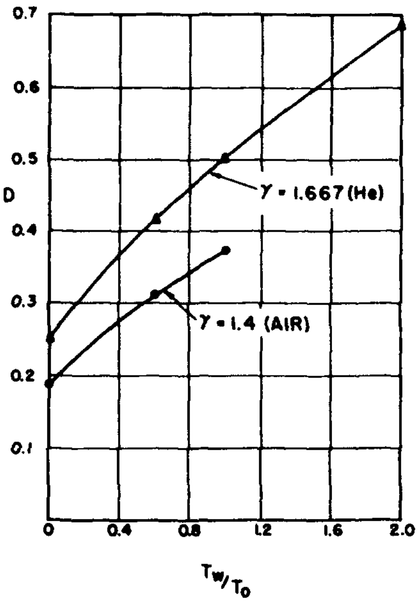
1. Values of A (Eq. 16) vs. T_W/T_O, γ = 1.4 and γ = 1.67



2. Values of B (Eq. 17) vs. T_W/T_O, γ = 1.4 and γ = 1.67

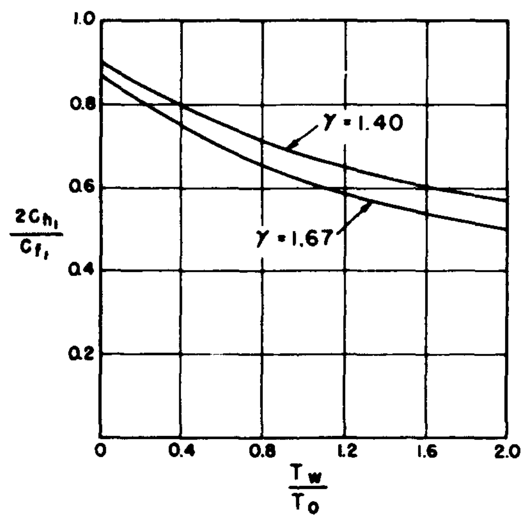


3. Values of C (Eq. 18) vs. T_w/T_o, $\gamma = 1.4$ and $\gamma = 1.67$



4. Values of D (Eq. 19) vs. T_w/T_o, $\gamma = 1.4$ and $\gamma = 1.67$

heated surface. From these figures, it can readily be concluded that cooling the surface will decrease the strong interaction effects between the shock wave and the boundary layer. The effects of cooling are many fold: (1) cooling the surface tends to thin down the boundary layer thickness, (Fig. 2); (2) cooling the surface tends to induce a smaller pressure rise in the interaction zone (Fig. 1); (3) cooling the surface tends to lower the skin friction coefficient for a fixed Re_x (Fig. 3). The values of $2C_{h_\infty}/C_{f_\infty}$ for various T_w/T_0's are presented in Fig. 5. This parameter tends to increase when T_w/T_0 decreases to zero. The effects of heating the surface are just the opposite of the above.

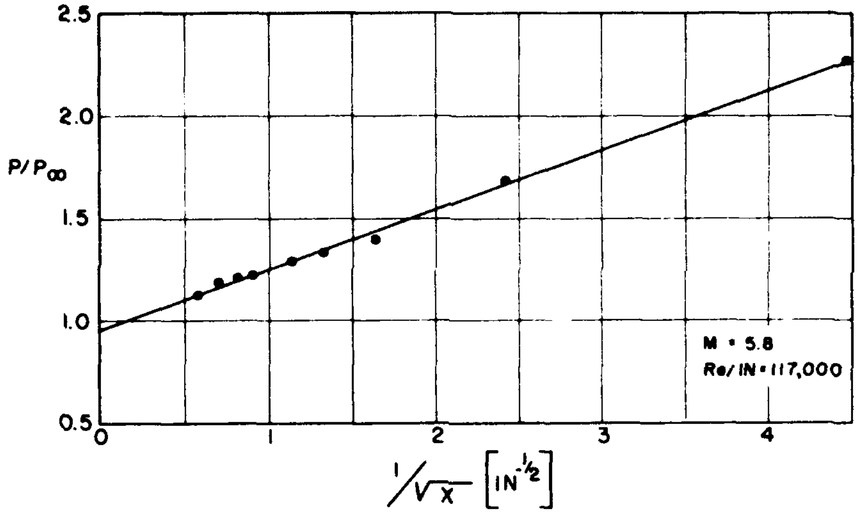


5. $2Ch_\infty/Cf_\infty$ vs. T_w/T_0, $\gamma = 1.4$ and $\gamma = 1.67$

Discussion

The recent experimental findings at GALCIT [12] confirm that

(1) the static pressure on the plate surface varies as $x^{-1/2}$ (Fig. 6)



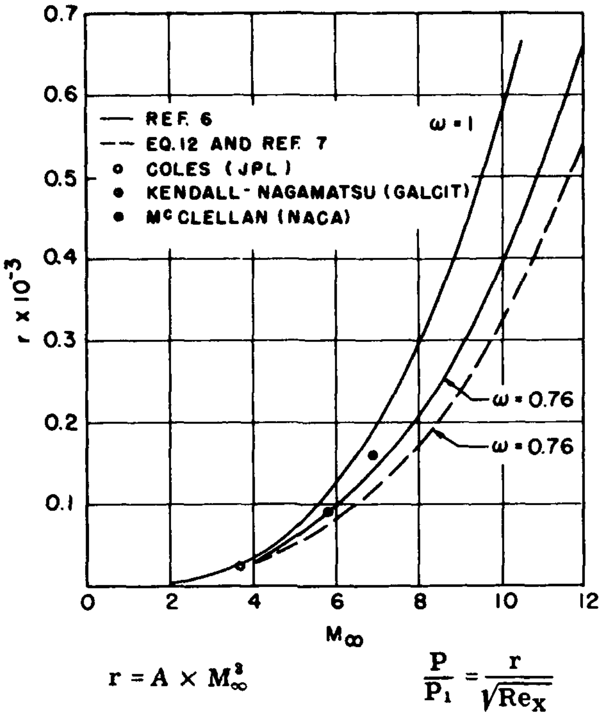
6. Static Pressure Distrubution on a Flat Plate.

(2) the velocity at the edge of the boundary layer is nearly constant: $u_1 \simeq u_\infty$

On the basis of these facts, a "similar" solution can be found for the problem of the hypersonic strong interaction between the shock wave and the boundary layer on a flat plate. More extensive experimental studies are necessary to verify the theoretical predictions of the behavior of the boundary layer skin friction and heat transfer characteristics.

At the present time, in the particular case of an insulated flat plate, a comparison of the values of A from experiment and from theory can be made. This comparison is shown in Fig. 7. It is seen that the air data at $M_\infty = 5.8$ obtained in the GALCIT Hypersonic Wind Tunnel, Leg No. 1, seem to agree quite well with the theories. It is also interesting to note that the agreement with the theory in [6] seems even better than the present computations. This is perhaps due to two inaccuracies in the present analysis:

(1) the assumed linear variation of μ with T,
(2) the approximate matching of the viscous and inviscid regions by the tangent-wedge approximation.



7. Comparison of Theories and Experiments.

In [6], of course, the more accurate variation of μ with T is used, namely, $\mu \propto T^{\omega}$, with the result that the hypersonic parameter, $M_\infty^{2+\omega}/\sqrt{Re_x}$, is obtained instead of the more restrictive parameter, $M_\infty^3/\sqrt{Re_x}$. Stewartson, in a recent study [15] of the hypersonic strong interaction phenomena, assumed that a "similar" solution also exists for the inviscid rotational flow region between the shock wave and the boundary layer. Thus, he was able to affect a matching at the edge of the boundary layer without using the tangent-wedge approximation. With Stewartson's method of matching, the values of A would come closer to the experimental values.

The helium data at M_∞ = 12.7 obtained in the Princeton tunnel [13] give considerably larger values of A than the

theoretical predictions, which might be due to the fact that the flat plate model used at Princeton has a larger leading edge radius. The importance of the effects of the bluntness of the leading edge has been recently brought out by Bertram [16] and Lees [17]. As pointed out in [17], the present "similar" solutions probably represent the correct limiting forms of the solution for the case of an ideally sharp leading edge plate when the hypersonic parameter, $M_\infty^3/\sqrt{Re_x}$, is held constant as $M_\infty \rightarrow \infty$.

In this idealized situation, the consideration of the bluntness of the leading edge and the estimation of the effects of the entropy layer from the leading edge region are not possible. To remedy this unrealistic omission, it is necessary to study the leading edge region more carefully.

At the present, nothing can be said about the comparison of experiment and theory for the case of hypersonic strong interaction on a noninsulated flat plate. Extensive experimental studies in this direction should be initiated. Nevertheless, by inference from the present results, it is felt that the hypersonic strong interaction phenomena can be reasonably controlled by providing sufficient surface cooling.

REFERENCES

1. Shen, S. F., An Estimate of Viscosity Influence on Hypersonic Flow over an Insulated Wedge, Jour. Math. Phys., Vol. 31, No. 3, 1952.
2. Shen, S. F., On the Boundary Layer Equations in Hypersonic Flow, Jour. Aero. Sci., Vol. 19, No. 7, p. 500, July 1952.
3. Becker, J. V., Results of Recent Hypersonic and Unsteady Flow Research at the Langley Aeronautical Laboratory, Jour. Appl. Phys., Vol. 21, No. 7, pp. 619-628, July 1950.
4. Bertram, M. H., An Approximate Method for Determining the Displacement Effects and Viscous Drag of Laminar Boundary Layers in Two-Dimensional Hypersonic Flow, NACA TN 2773, 1952.
5. Lees, L. and Probstein, R. F., Hypersonic Viscous Flow over a Flat Plate, Princeton University Aeronautical Engineering Laboratory Report No. 195, 1952.
6. Li, T. Y. and Nagamatsu, H. T., Shock-Wave Effects on the Laminar Skin Friction of an Insulated Flat Plate at Hypersonic Speeds, Jour. Aero. Sci., Vol. 20, No. 5, pp. 345-355, May 1953; also GALCIT Hypersonic Wind Tunnel Report, Memorandum No. 9, 1952.
7. Lees, L., On the Boundary-Layer Equations in Hypersonic Flow and Their Approximate Solutions, Jour. Aero. Sci., Vol. 20, No. 2, p. 143, Feb. 1953.
8. Lees, L. and Probstein, R. F., Hypersonic Flows of a Viscous Fluid, (unpublished).

9. Stewartson, K., Correlated Incompressible and Compressible Boundary Layers, Proc. Roy. Soc., London, Vol. 200, pp. 84-100, 1949.

10. Li, T. Y. and Nagamatsu, H. T., Similar Solutions of Compressible Boundary Layer Equations, Jour. Aero. Sci., Vol. 20, No. 9, p. 653, Sept. 1953.

11. Li, T. Y. and Nagamatsu, H. T., Similar Solutions of Compressible Boundary Layer Equations, Proc. Heat Transfer and Fluid Mechanics Institute, pp. 143-157, 1954, also GALCIT Hypersonic Wind Tunnel Report, Memorandum No. 22, 1954.

12. Nagamatsu, H. T., Summary of Recent GALCIT Hypersonic Experimental Investigations, Jour. Aero. Sci., Vol. 22, No. 3, p. 165, March 1955.

13. Hammitt, A. G. and Bogdonoff, S. M., A Study of the Flow about Simple Bodies at Mach Numbers from 11 to 15, Princeton University Aeronautical Engineering Laboratory Report No. 277, Sept. 1954.

14. Li, T. Y. and Nagamatsu, H. T., Hypersonic Viscous Flow on a Noninsulated Flat Plate, GALCIT Hypersonic Wind Tunnel Report, Memorandum No. 25, April 1955.

15. Stewartson, K., On the Motion of a Flat Plate at High Speed in a Viscous Compressible Fluid—II. Steady Motion, Jour. Aero. Sci., Vol. 22, No. 5, p. 303, May 1955.

16. Bertram, M. H., Viscous and Leading Edge Thickness Effects on the Pressures on the Surface of a Flat Plate in Hypersonic Flow, Jour. Aero. Sci., Vol. 21, No. 6, p. 430, June 1954.

17. Lees, L., Influence of the Leading Edge Shock Wave on the Laminar Boundary Layer at Hypersonic Speeds, GALCIT TR No. 1, July 1954.